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Quantum dynamical $\check{R}$-matrix with spectral parameter from fusion

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Abstract. A quantum dynamical $\check{R}$-matrix with a spectral parameter is constructed by fusion procedure. This spin-1 $\check{R}$-matrix is connected with Lie algebra $so(3)$ and does not satisfy the condition of translation invariance.

1. Introduction

Since the classical dynamical r-matrix [1] first appeared on the scene of integrable many-body systems, many dynamical r-matrices have been found in integrable models such as the Calogero–Moser model [2], the sine-Gordon soliton case [3] and the general case for the Ruijsenaars system [4]. These dynamical r-matrices do not satisfy the ordinary classical Yang–Baxter equation, so their quantization is rather non-trivial. The quantum dynamical Yang–Baxter (QDYB) equation, which first appeared in the quantization of Toda field theory [5] and later in the quantization of the KZB (Knizhnik–Zamolodchikov–Bernard) equation [6], had been studied widely for various integrable models and its algebraic structure was explored [7–9].

In contrast to the non-dynamical one [10], only a few dynamical R-matrices are constructed explicitly and most of them can be obtained from Felder's solution [6] by taking a gauge transformations [9]. So how to construct new R-matrix is still an interesting and challenging problem. As an efficient method to obtain a higher-spin R-matrix, fusion procedure [11] has been applied to the dynamical R-matrix [12].

In this paper, we construct a spin-1 quantum dynamical R-matrix with spectral parameter by 'fusing' together the spin-$\frac{1}{2}$ R-matrices which satisfy the QDYB equation [7]:

$$
\begin{aligned}
&R_{12}(\lambda_{12}, x+\gamma h^{(3)})R_{13}(\lambda_{13}, x-\gamma h^{(2)})R_{23}(\lambda_{23}, x+\gamma h^{(1)}) \\
&\qquad = R_{23}(\lambda_{23}, x-\gamma h^{(1)})R_{13}(\lambda_{13}, x+\gamma h^{(2)})R_{12}(\lambda_{12}, x-\gamma h^{(3)}).
\end{aligned}
\tag{1}
$$

The spectral parameters λ_{ij} are defined as $\lambda_{ij}=\lambda_i-\lambda_j$, $x=\sum_\nu x_\nu h_\nu$ is the dynamical variable and h is the Cartan subalgebra of the underlying simple Lie algebra. Taking values in $\mathrm{End}(V_1\otimes V_2\otimes V_3)$, the R-matrix appears as $R_{12}(x+\gamma h^{(3)})(V_1\otimes V_2\otimes V_3)=(R_{12}(x+\gamma\mu)(V_1\otimes V_2))\otimes V_3$ if $h^{(3)}$ has weight μ in space V_3. Other symbols have a similar meaning.

In braid form, the QDYB equation (1) reads as

$$\check{R}_{23}(\lambda_{12}, x+\gamma h^{(1)})\check{R}_{12}(\lambda_{13}, x-\gamma h^{(3)})\check{R}_{23}(\lambda_{23}, x+\gamma h^{(1)}) \\ = \check{R}_{12}(\lambda_{23}, x-\gamma h^{(3)})\check{R}_{23}(\lambda_{13}, x+\gamma h^{(1)})\check{R}_{12}(\lambda_{12}, x-\gamma h^{(3)}) \tag{2}$$

where $\check{R}_{ij} = P_{ij}R_{ij}$ and P_{ij} is the permutation operator acting on spaces $V_i \otimes V_j$. If $\check{R}$-matrices satisfy the condition of translation invariance:

$$[\mathcal{D}^{(i)} + \mathcal{D}^{(j)}, \check{R}_{ij}(\lambda, x)] = 0 \qquad \mathcal{D}^{(i)} = \sum_{\nu} h_{\nu}^{(i)}\partial_{x_\nu} \tag{3}$$

we can rewrite equation (2) as

$$\check{R}_{23}(\lambda_{12}, x+2\gamma h^{(1)})\check{R}_{12}(\lambda_{13}, x)\check{R}_{23}(\lambda_{23}, x+2\gamma h^{(1)}) \\ = \check{R}_{12}(\lambda_{23}, x)\check{R}_{23}(\lambda_{13}, x+2\gamma h^{(1)})\check{R}_{12}(\lambda_{12}, x). \tag{4}$$

This paper is organized as follows. In section 2, we obtain some useful properties of the $\check{R}^{(\frac{1}{2},\frac{1}{2})}$-matrix. In section 3, using the $\check{R}^{(\frac{1}{2},\frac{1}{2})}$-matrix, we construct the $\check{R}^{(1,1)}$-matrix by fusion procedure and prove that the new matrix also satisfies the QDYB equation. Finally, we discuss our results and compare them with [12] in section 4.

2. Properties of the spin-$\frac{1}{2}$ $\check{R}$-matrix

According to spin-$\frac{1}{2}$ chain, $h^{(i)}(\otimes V_i) = \mathrm{diag}\{\frac{1}{2}, -\frac{1}{2}\}(\otimes V_i)$, there is the simplest $\check{R}$-matrix solution with spectral parameter [7]:

$$\check{R}^{(\frac{1}{2},\frac{1}{2})}(\lambda, x) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{\sinh\gamma\sinh(x+\lambda)}{\sinh x\sinh(\lambda-\gamma)} & \frac{\sinh\lambda\sinh(x+\gamma)}{\sinh x\sinh(\lambda-\gamma)} & 0 \\ 0 & \frac{\sinh\lambda\sinh(x-\gamma)}{\sinh x\sinh(\lambda-\gamma)} & -\frac{\sinh\gamma\sinh(x-\lambda)}{\sinh x\sinh(\lambda-\gamma)} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{5}$$

This $\check{R}^{(\frac{1}{2},\frac{1}{2})}$-matrix satisfies the 'weight zero' condition

$$[h^{(i)} + h^{(j)}, \check{R}_{ij}(\lambda, x)] = 0 \tag{6}$$

and it has one triple eigenvalue 1 and one single eigenvalue $-\frac{\sinh(\lambda+\gamma)}{\sinh(\lambda-\gamma)}$.

To the triple eigenvalue, its right-acting eigenvectors are

$$u_{(1)}(x) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad u_{(0)}(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad u_{(-1)}(x) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{7}$$

and its left-acting eigenvectors are

$$\overline{u}^{(1)}(x) = (1, 0, 0, 0) \\ \overline{u}^{(0)}(x) = \frac{1}{\sqrt{2}}\left(0, \frac{\sinh(x-\gamma)}{\sinh x\cosh\gamma}, \frac{\sinh(x+\gamma)}{\sinh x\cosh\gamma}, 0\right) \\ \overline{u}^{(-1)}(x) = (0, 0, 0, 1). \tag{8}$$

While the eigenvalue is $-\frac{\sinh(\lambda+\gamma)}{\sinh(\lambda-\gamma)}$, the right-acting and left-acting eigenvectors are

$$v_{(0)}(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \frac{\sinh(x+\gamma)}{\sinh x\cosh\gamma} \\ \frac{-\sinh(x-\gamma)}{\sinh x\cosh\gamma} \\ 0 \end{pmatrix} \qquad \overline{v}^{(0)}(x) = \frac{1}{\sqrt{2}}(0, 1, -1, 0) \tag{9}$$

respectively.

These eigenvectors satisfy

$$\begin{aligned} &\overline{u}^{(a)}(x)v_{(0)}(x) = \overline{v}^{(0)}(x)u_{(a)}(x) = 0 \qquad a = 1, 0, -1 \\ &\overline{v}^{(0)}(x)v_{(0)}(x) = 1 \qquad \overline{u}^{(a)}(x)u_{(b)}(x) = \delta^a_b \qquad a, b = 1, 0, -1 \end{aligned} \tag{10}$$

so we can construct two projection operators for the triplet and singlet

$$\begin{aligned} &P(x) = \sum_a u_{(a)}(x)\overline{u}^{(a)}(x) \qquad Q(x) = v_{(0)}(x)\overline{v}^{(0)}(x) \\ &\mathrm{id}_{(4\times4)} = P(x) + Q(x) \end{aligned} \tag{11}$$

in which $\mathrm{id}_{(4\times4)} = \mathrm{diag}\{1, 1, 1, 1\}$, $P(x)$ and $Q(x)$ have the properties:

$$\begin{aligned} &P^2(x) = P(x) \qquad Q^2(x) = Q(x) \qquad P(x)Q(x) = Q(x)P(x) = 0 \\ &P(x)u_{(a)}(x) = u_{(a)}(x) \qquad \overline{u}^{(a)}(x)P(x) = \overline{u}^{(a)}(x) \qquad a = 1, 0, -1. \end{aligned}$$

Now, we can rewrite $\check{R}^{(\frac{1}{2},\frac{1}{2})}(\lambda, x)$ as

$$\check{R}^{(\frac{1}{2},\frac{1}{2})}(\lambda, x) = P(x) - \frac{\sinh(\lambda+\gamma)}{\sinh(\lambda-\gamma)}Q(x).$$

It is obvious that

$$\check{R}^{(\frac{1}{2},\frac{1}{2})}(\lambda = -\gamma, x) = P(x). \tag{12}$$

Applying this property to equation (2), we obtain

$$\begin{aligned} &P_{23}(x+\gamma h^{(1)})\check{R}^{(\frac{1}{2},\frac{1}{2})}_{12}(\lambda-\gamma, x-\gamma h^{(3)})\check{R}^{(\frac{1}{2},\frac{1}{2})}_{23}(\lambda, x+\gamma h^{(1)}) \\ &\qquad = \check{R}^{(\frac{1}{2},\frac{1}{2})}_{12}(\lambda, x-\gamma h^{(3)})\check{R}^{(\frac{1}{2},\frac{1}{2})}_{23}(\lambda-\gamma, x+\gamma h^{(1)})P_{12}(x-\gamma h^{(3)}) \\ &\check{R}^{(\frac{1}{2},\frac{1}{2})}_{23}(\lambda, x+\gamma h^{(1)})\check{R}^{(\frac{1}{2},\frac{1}{2})}_{12}(\lambda-\gamma, x-\gamma h^{(3)})P_{23}(x+\gamma h^{(1)}) \\ &\qquad = P_{12}(x-\gamma h^{(3)})\check{R}^{(\frac{1}{2},\frac{1}{2})}_{23}(\lambda-\gamma, x+\gamma h^{(1)})\check{R}^{(\frac{1}{2},\frac{1}{2})}_{12}(\lambda, x-\gamma h^{(3)}). \end{aligned} \tag{13}$$

3. Construction of the spin-1 $\check{R}$-matrix

In reference to fusion procedures in [11, 12], we 'fuse' the dynamical $\check{R}^{(1,1)}$ matrix with a spectral parameter as follows:

$$\begin{aligned} [\check{R}^{(1,1)}_{12,34}(\lambda, x)]^{ab}_{cd} &= \overline{u}^{(a)}_{12}(x-\gamma h^{(3,4)})\overline{u}^{(b)}_{34}(x+\gamma h^{(1,2)})\check{R}^{(\frac{1}{2},\frac{1}{2})}_{23}(\lambda+\gamma, x+\gamma h^{(1)}-\gamma h^{(4)}) \\ &\quad \times\check{R}^{(\frac{1}{2},\frac{1}{2})}_{12}(\lambda, x-\gamma h^{(3,4)})\check{R}^{(\frac{1}{2},\frac{1}{2})}_{34}(\lambda, x+\gamma h^{(1,2)}) \\ &\quad \times\check{R}^{(\frac{1}{2},\frac{1}{2})}_{23}(\lambda-\gamma, x+\gamma h^{(1)}-\gamma h^{(4)})u_{12(c)}(x-\gamma h^{(3,4)}) \\ &\quad \times u_{34(d)}(x+\gamma h^{(1,2)}) \end{aligned} \tag{14}$$

in which a, b, c, d take values among $1, 0, -1$ and $h^{(i,j)}$ means $h^{(i)} + h^{(j)}$, so this $\check{R}^{(1,1)}$ matrix is a 9×9 matrix.

In order to prove that equation (14) also satisfies the QDYB equation, we define two 4×4 matrices as follows:

$$u = (u_{(1)}, u_{(0)}, 0, u_{(-1)}) \qquad \overline{u} = \begin{pmatrix} \overline{u}^{(1)} \\ \overline{u}^{(0)} \\ 0 \\ \overline{u}^{(-1)} \end{pmatrix}.$$

We then replace $\overline{u}^{(a)}$ and $\overline{u}^{(b)}$ by $\overline{u}$ as well as replacing $u_{(c)}$ and $u_{(d)}$ by u in equation (14), such that $\check{R}^{(1,1)}$ is changed into a 16×16 matrix, where the added seven rows and seven columns are in fact nothing but zero. Such u and $\overline{u}$ matrices not only keep $u(x)\overline{u}(x) = P(x)$, $P(x)u(x) = u(x)$ and $\overline{u}(x)P(x) = \overline{u}(x)$, but also satisfy the weight zero condition too. Now the QDYB equation becomes

$$\begin{aligned}\check{R}^{(1,1)}_{34,56}(\lambda_{12}, x + \gamma h^{(1,2)})\check{R}^{(1,1)}_{12,34}(\lambda_{13}, x - \gamma h^{(5,6)})\check{R}^{(1,1)}_{34,56}(\lambda_{23}, x + \gamma h^{(1,2)})\\ = \check{R}^{(1,1)}_{12,34}(\lambda_{23}, x - \gamma h^{(5,6)})\check{R}^{(1,1)}_{34,56}(\lambda_{13}, x + \gamma h^{(1,2)})\check{R}^{(1,1)}_{12,34}(\lambda_{12}, x - \gamma h^{(5,6)}).\end{aligned} \quad (15)$$

For simplicity, we introduce $\check{\mathcal{R}}_{ij}(\lambda) := \check{R}^{(\frac{1}{2},\frac{1}{2})}_{ij}(\lambda, x + \gamma \sum_{k=1}^{i-1} h^{(k)} - \gamma \sum_{l=j+1}^{6} h^{(l)})$, and replace $u_{ij}(x + \gamma \sum_{k=1}^{i-1} h^{(k)} - \gamma \sum_{l=j+1}^{6} h^{(l)})$ and $\overline{u}_{ij}(x + \gamma \sum_{k=1}^{i-1} h^{(k)} - \gamma \sum_{l=j+1}^{6} h^{(l)})$ by $\sqcap_{ij}$ and $\overline{\sqcap}_{ij}$ respectively. After these notations, the weight zero condition means

$$[A_{ii+1}(\lambda), B_{jj+1}(\lambda')] = 0 \qquad \text{if } i + 1 < j \text{ or } j + 1 < i \quad (16)$$

in which $A, B \in \{\check{\mathcal{R}}, \sqcap, \overline{\sqcap}\}$. By the relation (13) and its analogue, we can reduce equation (15) to

$$\begin{aligned}&\text{l.h.s.} = \overline{\sqcap}_{12}\overline{\sqcap}_{34}\overline{\sqcap}_{56}S_{34}(\lambda_{12})S_{12}(\lambda_{13})S_{34}(\lambda_{23})\sqcap_{12}\sqcap_{34}\sqcap_{56}\\ &\text{r.h.s.} = \overline{\sqcap}_{12}\overline{\sqcap}_{34}\overline{\sqcap}_{56}S_{12}(\lambda_{23})S_{34}(\lambda_{13})S_{12}(\lambda_{12})\sqcap_{12}\sqcap_{34}\sqcap_{56}\\ &S_{ii+1}(\lambda) = (\check{\mathcal{R}}_{i+1i+2}(\lambda - \gamma)\check{\mathcal{R}}_{ii+1}(\lambda)\check{\mathcal{R}}_{i+2i+3}(\lambda)\check{\mathcal{R}}_{i+1i+2}(\lambda + \gamma)).\end{aligned}$$

Using the QDYB equation (2) and its analogue, we have proved $S_{34}(\lambda_{12})S_{12}(\lambda_{13})S_{34}(\lambda_{23}) = S_{12}(\lambda_{23})S_{34}(\lambda_{13})S_{12}(\lambda_{12})$, or l.h.s. = r.h.s. in the above equation. In other words, the fusion procedure is practicable.

If we rewrite equation (15) in the standard 9×9 matrix form $\check{R}^{(1,1)}_{IJ}(\lambda, x)$, it becomes

$$\check{R}^{(1,1)}_{JK}(\lambda_{12}, x + \gamma h^{(I)})\check{R}^{(1,1)}_{IJ}(\lambda_{13}, x - \gamma h^{(K)})\check{R}^{(1,1)}_{JK}(\lambda_{23}, x + \gamma h^{(I)}) \quad (17)$$

$$= \check{R}^{(1,1)}_{IJ}(\lambda_{23}, x - \gamma h^{(K)})\check{R}^{(1,1)}_{JK}(\lambda_{13}, x + \gamma h^{(I)})\check{R}^{(1,1)}_{IJ}(\lambda_{12}, x - \gamma h^{(K)}). \quad (18)$$

It is simply the original QDYB equation (2). Notice that this $\check{R}^{(1,1)}$ matrix is of spin-1 since $h^{(l)}(\otimes V_l)$ (in which $l \in \{I, J, K\}$) becomes $\mathrm{diag}\{1, 0, -1\}(\otimes V_l)$ by taking the singlet of spin-0 away.

With the $\check{R}^{(\frac{1}{2},\frac{1}{2})}$-matrix (5) and the fusion method (14), we obtain

$$\check{R}^{(1,1)}(\lambda, x) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & a(\lambda,x) & 0 & b(\lambda,-x) & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & c(\lambda,x) & 0 & d(\lambda,x) & 0 & e(\lambda,x) & 0 & 0\\ 0 & b(\lambda,x) & 0 & a(\lambda,-x) & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & f(\lambda,x) & 0 & g(\lambda,x) & 0 & f(\lambda,-x) & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & a(\lambda,x) & 0 & b(\lambda,-x) & 0\\ 0 & 0 & e(\lambda,-x) & 0 & d(\lambda,-x) & 0 & c(\lambda,-x) & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & b(\lambda,x) & 0 & a(\lambda,-x) & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (19)$$

in which

$$a(\lambda, x) = \frac{\sinh(2\gamma)\sinh(\lambda + x)}{\sinh(2\gamma - \lambda)\sinh x} \qquad b(\lambda, x) = \frac{\sinh(\lambda)\sinh(2\gamma - x)}{\sinh(2\gamma - \lambda)\sinh x}$$

$$c(\lambda, x) = \frac{\sinh\gamma\sinh(2\gamma)\sinh(\lambda + x)\sinh(\gamma + \lambda + x)}{\sinh(\gamma - \lambda)\sinh(2\gamma - \lambda)\sinh x\sinh(\gamma + x)}$$

$$d(\lambda, x) = \frac{\sinh(2\gamma)\sinh(\lambda)\sinh(2\gamma + x)\sinh(\lambda + x)\cosh\gamma}{\sinh(\gamma - \lambda)\sinh(2\gamma - \lambda)\sinh(\gamma - x)\sinh(\gamma + x)}$$

$$e(\lambda, x) = -\frac{\sinh\lambda\sinh(\gamma + \lambda)\sinh(\gamma + x)\sinh(2\gamma + x)}{\sinh(\gamma - \lambda)\sinh(2\gamma - \lambda)\sinh(\gamma - x)\sinh x}$$

$$f(\lambda, x) = \frac{2\sinh\gamma\sinh\lambda\sinh(\gamma - x)\sinh(\lambda + x)}{\sinh(\gamma - \lambda)\sinh(2\gamma - \lambda)\sinh x\sinh(\gamma + x)}$$

$$g(\lambda, x) = \frac{\sinh(\gamma + \lambda)}{\sinh(\gamma - \lambda)} + \frac{\sinh\lambda(\cosh(2x) - \cosh(2\gamma) - \sinh^2(2\gamma))}{\sinh(2\gamma - \lambda)\sinh(\gamma - x)\sinh(\gamma + x)}.$$

The obtained $\check{R}^{(1,1)}$-matrix has three distinct eigenvalues, say, 1, $-\frac{\sinh(\lambda+2\gamma)}{\sinh(\lambda-2\gamma)}$ and $\frac{\sinh(\lambda+\gamma)\sinh(\lambda+2\gamma)}{\sinh(\lambda-\gamma)\sinh(\lambda-2\gamma)}$ whose multiplicities are 5, 3 and 1 respectively. This $\check{R}^{(1,1)}$ is connected with Lie algebra $so(3)$. By direct calculation, we can show that it does satisfy the QDYB equation (17) with $h^{(l)}(\otimes V_l) = \mathrm{diag}\{1, 0, -1\}(\otimes V_l)$.

4. Discussion

From expression (18), we find that the $\check{R}^{(1,1)}$-matrix does not satisfy the translation invariance condition (3). In other words, if we want to translate it to the form of equation (4), we will obtain a more complex $\check{R}^{(1,1)}$-matrix form. In fact, it is just the matrix of $\check{R}^{(1,1)}_{IJ}(\lambda, x+\gamma h^{(I,J)})$ where $h^{(I,J)}$ means $h^{(I)}+h^{(J)}$, so we have to construct new commuting operators different from those in [7], in which condition (3) was used in constructing commuting operators. For simplicity of the expression of the R-matrix, we had better use a more symmetric form as in equation (1) or (2), rather than the form as in equation (4).

We now compare our results with [12]. First, the QDYB equation (4) tends to be independent of the spectral parameter by requiring $\lambda \to \pm\infty$. Secondly, we need to change the dynamical variable $x \to -\gamma x$ in our $\check{R}^{(\frac{1}{2},\frac{1}{2})}$- and $\check{R}^{(1,1)}$-matrices because the QDYB equation takes different forms in these two papers. Finally, we need to translate expression (18) to $\check{R}^{(1,1)}_{IJ}(\lambda, x+\gamma h^{(I,J)})$ as discussed before. After these changes of λ and x in $\check{R}^{(1,1)}_{IJ}(\lambda, x+\gamma h^{(I,J)})$, we indeed obtain the $\check{R}^{(1,1)}$-matrix gauge equivalent to the one in [12]. The single eigenvalue q of the $\check{R}^{(\frac{1}{2},\frac{1}{2})}$-matrix in [12] is connected to $\mathrm{e}^{\pm 2\gamma}$ when we take $\lambda \to \pm\infty$, respectively.

For the six-vertex elliptic solution of the QDYB equation, eigenvalues of the $\check{R}^{(\frac{1}{2},\frac{1}{2})}$-matrix are not made of one triplet and one singlet. It is still an open problem concerning how to construct its higher-spin $\check{R}$-matrix.

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References

[1] Avan J and Talon M 1993 *Phys. Lett.* B **303** 33
[2] Sklyanin E K 1994 *Alg. Anal* **6** 227 (hep-th/9308060)
Braden H W and Suzuki T 1994 *Lett. Math. Phys.* **30** 147
Billey E, Avan J and Babelon O 1994 *Phys. Lett.* A **186** 114
[3] Babelon O and Bernard D 1993 *Phys. Lett.* B **317** 363
[4] Avan J and Rollet G 1995 The classical r-matrix for the relativistic Ruijsenaars–Schneider system *Preprint* Browm HET 1014, hep-th/9510166
Suris Y B 1997 *Phys. Lett.* A **225** 253
[5] Gervais J L and Neveu A 1984 *Nucl. Phys.* B **238** 125

[6] Felder G 1994 Conformal field theory and integrable systems associated to elliptic curve *(Proc. ICM94, Birkhauser) Preprint* hep-th/9407154
Felder G 1994 Elliptic quantum groups *Proc. ICMP (Paris)*, hep-th/9412207
[7] Avan J, Babelon O and Billey E 1996 *Commun. Math. Phys.* **178** 281
[8] Arnaudon D, Buffenoir E, Ragoucy E and Roche Ph 1998 *Lett. Math. Phys.* **44** 201
Babelon O, Bernard D and Billy E 1996 *Phys. Lett.* B **375** 89
Felder G, Tarasov V and Varchenko A 1996 Solutions of the elliptic qKZB equations and Bethe ansatz I *Preprint* q-alg/9606005
[9] Etingof P and Varchenko A 1997 Solutions of the quantum dynamical Yang–Baxter equation and dynamical quantum groups *Preprint* q-alg/9708015
Arutyunov G E and Frolov S A 1998 *Commun. Math. Phys.* **191** 15
[10] Ma Z Q 1993 *Yang–Baxter Equation and Quantum Enveloping Algebras* (Singapore: World Scientific) p 1
Sun X D, Wang S K and Wu K 1995 *J. Math. Phys.* **36** 6043
Wang S K 1996 *J. Phys. A: Math. Gen.* **29** 2259
[11] Kulish P P and Sklyanin E K 1982 (*Lecture Notes in Physics 151*) (New York: Springer) p 61
Kulish P P, Reshetikhin N Y and Sklyanin E K 1981 *Lett. Math. Phys.* **5** 393
Mezincescu L and Nepomechie R I 1991 Quantum groups *Proc. Argenne Workshop* (Singapore: World Scientific) p 206
[12] Song X C 1998 Quantum dynamical R-matrix from fusion *Commun. Theor. Phys.* **30** 157